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The acceptability of papers offered must largely depend upon the judgments of our editorial referees. Final decisions will be made as promptly as possible and reported to those concerned.

This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

Back-ground in Mathematics

The understanding of a new set of ideas or a new mechanism must be acquired through comparison with the individual's previous knowledge of related ideas or mechanisms. This has been called the Principle of Apperception. As a corollary to this pedagogical principle we have: In beginning the presentation of a new subject (which is a new set of ideas), it is essential that the students have a suitable back-ground for assimilating and understanding the new material.

A second corollary is: It is the duty of the teacher to determine whether the pupils have this back-ground and to make provision for it when necessary. In this way a good teacher may render superior service.

It is in respect to back-ground that much of the teaching of Euclid and formal algebra has been unsatisfactory in the past. The ideas of space held by average young minds at the beginning of the study of geometry are very different from those needed to understand formal geometry. Euclid's treatise was prepared for mature men of experience. Some provision must be made for the initial stages. In many schools this is done. A triangular park or piece of paper, or a triangular figure on the black board, is quite different from the abstract invisible aggregate of triangles in space and symbolized by conventional diagrams. A similar truth holds with respect to formal algebra. The symbol x, used to represent any one of a vast aggregate of individual numbers is quite different from the concept of an isolated number. Understanding of the symbols x and a as representing any numbers of a set, is difficult as compared to the concept of individual numbers such as 2, 3, 2/3, etc.

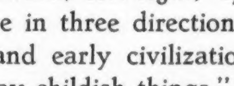
The new emphasis on laws and processes as compared with the previous emphasis on numerical results presents difficulty. Suitable back-ground in mathematics can hardly be overestimated. Fundamentals must be understood if they are to carry over to the generalized cases. —W. PAUL WEBBER.

SPACE*

By Dorothy McCoy
Belhaven College

Space, backward-forward, left-right, up-down as far as we can conceive, is space: space in three directions, three dimensional space. But this is a childish and early civilization viewpoint and "when I became a man I put away childish things." Space then has grown up--no not space but what we choose the word to mean has changed, is ever changing and means different things to different people. We shall look about and discover what we discover.

With plane geometry and the advent of planes and lines, we observe that a plane has two directions or two dimensions, a line has one dimension, and then with one more step we call a point zero-dimensional. So we have zero, one, two and three dimensional spaces, and already the word is plural. Why stop with three dimensions? Well, there appears no other direction to go and in case there were, just what could we find in that new direction? First with high school geometry for a background we shali look about, then, a little later, assume a greater mathematical preparation.



Take a look at two symmetrical triangles in a plane, having corresponding parts equal; then try sliding one about to make it fit on top of the other without moving it off the plane or changing its shape Yes, that is impossible, but if it is picked up into the third dimension, it turns over readily and we find the two triangles coincide perfectly. Now consider your two hands; they are symmetrical figures in three dimensions and we know from trying the right glove on the left hand how impossible it is to make them coincide. We need a fourth direction, a new freedom, in which to move in order to bring three dimensional figures into coincidence just as our two dimensional triangles had to move in a third direction to be superimposed.

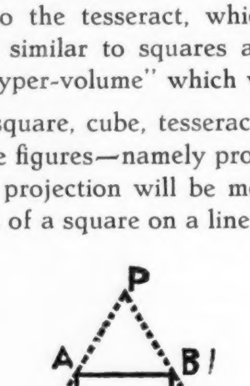
Where we find properties common to the lower dimensions we, by analogy, extend them to a conceived four dimensional space. For

* This paper is written for the undergraduate student of mathematics, especially for those who can handle more than the average. The facts in it I use regularly: (1) to present the generalization of analytic geometry to more dimensions and (2) to arouse and maintain interest in mathematics.

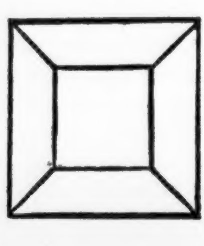
example, three equidistant points in a plane form an equilateral triangle. A number of such triangles fit together perfectly so as to cover any connected portion of the plane. Similarly four equidistant points in three dimensions (call it 3-space) form a tetrahedron, also a space filling configuration. Then five equidistant points in four dimensions (4-space) would be conceived to pack without leaving any holes and thus fill 4-space.

In much the same way the notion of area to volume can be extended. The unit of area is a square and of volume a cube. The square has four vertices and area a^2; the cube has eight vertices and volume a^3. Extending to the tesseract, which is a four dimensional figure having properties similar to squares and cubes, we see that it has 16 vertices and a "hyper-volume" which we would represent by a^4.

From this sequence-square, cube, tesseract—we can illustrate other ways of studying 4-space figures—namely projection and folding down. Perhaps the analogy of projection will be more easily extended if we go back to the projection of a square on a line from a point in the plane.



Looking from the point P the line AB coincides with EF, the line BC goes into CF, etc., that is, the square ABCD projects into the figure EFCD in one dimension. We note that this is a line segment DC within a line segment EF. Similarly, looking at a cube from a point directly above and projecting to the plane on which it rests we see a square within a square. That is, the two dimensional configuration—



the square—being bounded by lines, projects into one segment within another; the three dimensional cube, being bounded by squares, projects into the lower space as a square within a square. Hence the tesseract would be bounded by cubes and project into 3-space as a cube within a cube. Our kodak pictures form good illustrations of projections of three dimensional objects on two dimensional space.

By projection then, we look at our configuration as of space of one lower dimension. This is likewise true when we "fold down" a figure. A square of wire can be straightened into a line by cutting it at one point. Cutting a cube at a number of its edges, lines, it folds down into six squares in a plane. We have cut first a point then a line hence, cutting a tesseract in the proper plane, it folds down into eight cubes in 3-space.

From the illustrations given it is seen that we conceive of a similar relation between 3-space and 4-space as we find between 2-space and 3-space. From this point of view consider a two dimensional intelligent being studying a square. This being crawling around the square would arrive at the conclusion that the four distances between vertices were equal. Now suppose we, as three dimensional beings, distort this square so that two adjacent vertices are almost together and we can see that it is no longer a square. Our two dimensional creature would not discover such a change in a third, to him impossible, direction. Similarly, curvature in a fourth direction would not be obvious to us as three dimensional beings.

With analytic geometry as a foundation for study of space, many concepts are easier to grasp. As column and row locates any required position on a page so two coordinates locate a point in the plane; that is, to two coordinates (x,y) corresponds a point in two dimensions to three coordinates (x,y,z) corresponds a point in three dimensions and four coordinates (x,y,z,w) is then the algebraic expression for a point in four dimensions. We are considering the number of dimensions of a space to be the number of coordinates necessary to locate a point of it. Truly this fourth axis of coordinates perpendicular to the other three cannot be visualized, yet the abstraction is not necessarily wrong as we cannot visualize "-3" apples, and negative numbers are thoroughly justified by their value in algebra.

In two dimensions two points (x_1,y_1), (x_2,y_2) have a distance given by the formula

$$[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2}$$

In 3-space the two points are (x_1,y_1,z_1), (x_2,y_2,z_2) and the formula becomes

$$[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2]^{1/2}$$

Now when one more coordinate is added this formula extends to

$$[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2+(w_2-w_1)^2]^{1/2}$$

as the distance between two points in 4-space. We find also that the distance from the plane $lx+my+nz-p=o$ to any point (x_1,y_1,z_1) is $lx_1+my_1+nz_1-p=d$ and that this is a direct extension of distance derived from the normal equation of a line in two dimensions. Extending the concept of direction cosine to a fourth axis, we have similarly a formula for distance from a hyperplane (linear equation represented in 4-spaces) to a point in four dimensions.

The formula for midpoint of a line segment between two points extends from

$$(x_1+x_2/2,\ y_1+y_2/2) \text{ to } (x_1+x_2/2,\ y_1+y_2/2,\ z_1+z_2/2)$$

and to $(x_1+x_2/2,\ y_1+y_2/2,\ z_1+z_2/2,\ w_1+w_2/2)$ as we advance

from two to three then to four dimensions. Many other algebraic expressions extend to four dimensions by analogy; for example, point dividing a segment in a given ratio, coordinates of centroid of a triangle, two point form of the equations of a line, equation of circle, etc.

In two dimensions, two linear equations representing two lines have a common solution which is the point of intersection of the two lines. Three linear equations—three planes in 3-space have a common point. Hence four linear equations—hyperplanes in 4-space—have a point in common.

Two linear equations in 2-space have a point in common; two linear equations in three space have a line in common; and we conclude two linear equations in 4-space have a plane in common. If in two equations of hyperplanes in 4-space; such as

$$2x+3y+4z-5w=4$$

$$x+\ y+\ z-\ w=1$$

we substitute values for two of the variables we can then determine the third and fourth uniquely, thus obtaining coordinates of points on their plane of intersection. Since we can choose any values for two variables, we have two degrees of freedom or two dimensions, a plane. This is a first extension of substituting for one variable and determining the second and third uniquely for two linear equations

in three unknowns thus obtaining coordinates of points on a line with one degree of freedom here. With similar reasoning, a line is given by one linear equation in 2-space, by two linear equations in 3-space and by 3 linear equations in 4-space.

As in two dimensions the four equations $x=0$, $x=1$, $y=0$, $y=1$ bound a square and the point $(1/2, 3/2)$ is a point suitable for projection of the square on the x-axis as described earlier, so in four dimensions we have the equations $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$ $w=0$, $w=1$ bounding a tesseract and the point $(1/2, 1/2, 1/2, 3/2)$ is a suitable one for projecting the tesseract on the xyz-space. The reader may interpolate for the case in 3-space and find the case in 4-space clearer. Coordinates of the vertices of the square, cube, and tesseract can be obtained and the various equations for determining the locus of the projected figure follow readily. This is one of the simpler cases of a projection into 3-space. Obviously any other point could be taken as the center of a projection and the tesseract studied from many angles.

Suppose now we consider the simplest of the second degree equations, that of a circle in 2-space, sphere in 3-space, and the so-called hypersphere in 4-space. The equation $x^2+y^2=1$ of the circle in 2-space becomes $x^2+y^2+z^2=1$ for a sphere in 3-space and $x^2+y^2+z^2+w^2=1$ for the hypersphere in 4-space. Analyzing the equation of the sphere analytically we find that every section of the sphere is a circle. That is, we have a new method of studying 3-space figures in terms of 2-space, by section or cutting through the figure with a 2-space or plane. As taking the special value $z=0$ in the equation of the sphere gives us a circle on the xy-plane, so by taking $w=0$ in the equation of the hypersphere we obtain a sphere in xyz-space. We see that every section of the hypersphere by a 3-space is a sphere. This is like slicing an apple to study it. Obviously this method of studying fourth dimensional configurations is quite helpful and can be used with any equation.

Now we are in a position to see something of what an "added direction" or a fourth axis may mean in number of figures possible in the space. There are three conic sections; that is, three types of second degree equations in two variables; there are more than double this number of quadric surfaces corresponding to the second degree equations in three variables, and still more possibilities with another variable added. The special cases of quadric surfaces are more numerous than of conic sections; likewise we could expect an even larger number of special cases in 4-space.

We have been discussing equations recently more than the pure geometry. This leads to mentioning that the use of geometrical language

in studying algebra is very helpful. This is especially true in considering systems of linear equations in many variables where the term dimension may be considered as a synonym for variable. By going back through this section it is obvious that the various extensions to four dimensions can be made just as readily to five or more dimensions; and having once passed the number three, where our ability to visualize ceased, and as the algebra extends more readily after taking one step there is no difficulty in speaking of n-dimensions, or n-space. Cayley (about 1844-1846) was the first to present the algebra of n-dimensional space. As suggested above the number of possible figures increases rapidly as we consider higher and higher dimensional spaces.

The physicist takes a different view of higher dimensional space, the most commonly discussed being that the fourth dimension is time. In general things differ in more respects than are indicated by the three conditions determining position; that is, things differ in such physical qualities as temperature, density, etc. A particular element is then determined by its coordinates and by certain other physical quantities, it is determined by a certain number of numerical parameters—frequently many more than three. The number of these parameters is called the dimension of the system.

From a somewhat different point of view physicists claim that the treatment of space as 3-dimensional limits it unnecessarily. From its study as more dimensional has come great changes in the theory of electromagnetic phenomena and a complete change in the laws of motion of rapidly moving bodies like electrons. As was suggested above, the physicist seems to find some evidence that our space is curved in a "fourth direction" which seems unthinkable to the ordinary mind.

The parameters the physicist uses in addition to those giving full position do not necessarily have all the meaning of an additional coordinate from the analytic point of view. A mathematical illustration of this comes from the consideration of the aggregate of lines in 3-space. Four independent variables are required to specify any one of them; hence the configuration may be called four dimensional although it does not constitute the usual three dimensions plus an added fourth.

So far we have thought of spaces as varying in regard to number of dimensions. Is it possible that the number of points in a space determines its dimension? Cantor has shown that it is possible to set up a correspondence between the points of a line and a plane in such a way that to a point of the line coresponds one and only one point of the plane and conversely. Then there are exactly the same number of

points on a line as on a plane, and by this scheme it would take only one parameter to locate a point on a plane by giving the parameter for its corresponding point on the line. Apparently then our concept of dimension involves more than we have hitherto considered as we seem to find a contradiction. Studying this correspondence established by Cantor we find that corresponding to two points close together on the line there may be two points of the plane very far apart. That is, the correspondence is not continuous. From this Frechet sets up a notion of dimension not so much dependent on the number of points involved as on the continuity of the correspondence established. Others have formulated various other definitions of dimension.

Besides continuity and dimension we usually conceive of space as having a property such that it is impossible to take just a point from it without taking at least some small portion of space with it. This portion of space related to the point is called the neighborhood of the point. Spaces developed with such a fundamental assumption are called neighborhood spaces.

Again we feel that between every two points of our sensed space there is a fixed distance. Abstracting from this, space is defined as an aggregate of points to every pair of points being given a fixed quantity called distance. If the points are given by expressions of the type (x,y) where x and y are real numbers the distance may be

$$[(x_2-x_1)^2+(y_2-y_1)^2]^{1/2} \text{ or } (x_2-x_1)(y_2-y_1)$$

or any other expression usually required to satisfy our most common intuitive notions of distance but not by any means agreeing with our analytical expression for distance. Taking different expressions for distance between points gives quite different types of space. Letting our imagination take flight, we might ask what would happen when Alice reaches for an apple if the distance between every two points of space were always greater than unity.

No attempt is being made to define all the types of space extensively studied either by mathematicians or physicists. Perhaps it would be well to look finally at a most general space called a topological space which is defined to consist of an aggregate of elements, usually called points, with some relation existing between the subaggregates. We note that in this space nothing is said about what the elements are—they could be chairs, lines, cows or Chinamen—and nothing is said regarding the type of relation existing among the groups. Also there is no requirement that space be infinite, continuous or have any property that we think of as dimension. It surely looks as though

this must be the final abstraction but after seeing other unexpected developments we can no longer rely on "obvious" for a conclusion.

We have gone from sensed space to topological space with rapid strides omitting much of interest and more that would be necessary should we try to reverse our steps in a thoroughly logical manner. Such a procedure is outside the range of this paper although we hope to have aroused a deeper interest in the properties of space.

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New Models for the Solution of Quadratic and Cubic Equations

By Arnold Emch
University of Illinois

In 1909 Roderich Hartenstein published a memoir on the "discriminant surface" of the equations of the fourth degree in explanation of a model constructed by the well known firm of Martin Schilling in Leipzig. It is based on the equation in the standard form

$$(1) \qquad t^4+6a_2t^2+4d_3t+d_4=0$$

Putting $d_2=x$, $d_3=y$, $d_4=z$, then with t as a variable parameter (1) represents a ∞' system of planes in a Cartesian (x,y,z)-space whose envelope is the discriminant surface. Treating a reduced cubic equation in a similar manner, Hartenstein first obtains the discriminant cuspidal plane cubic.

A year before Schilling announced a model of the discriminental surface for the quintic in the normal form

$$(2) \qquad u^5+10xu^3+5yu+z=0,$$

studied and prepared by Professor Emily Sinclair of Oberlin College. This work was suggested by Professor Oscar Bolza, then at Chicago, while Hartenstein's efforts were induced by Professor Felix Klein.

In what now follows I shall show that there is a much more effective and elegant method of representation, by models, for the solution of quadratic and cubic equations possible.

If x and y are coefficients and z the unknown, a quadratic equation may be written in the form

$$z^2+xz+y=0 \tag{3}$$

For every pair of values of x and y we get for z two values representing the roots z_1, z_2. Now consider x,y,z as coordinates of a Cartesian space, then (3) is an hyperbolic paraboloid whose generatrices are parallel to the xy-plane. The discriminant

$$x^2-4y=0 \tag{4}$$

is a parabolic cylinder which touches the quadric (3) along a conic. Every line l parallel to the z-axis passing through a point (x,y) cuts (3) in two points whose z-coordinates represent the two roots of the equation determined by the pair(x,y). For a line l within the parabolic cylinder (4) we obtain no real intersections with (3), i.e., imaginary roots. For a generatrix of (4) which touches the quadric (3) we obtain two coincident roots. When l is outside the parabolic cylinder it cuts the quadric (3) in two real points, i.e., there are two real roots.

A similar procedure may be followed in case of the normal form

$$z^3+xz+y=0 \tag{5}$$

for the cubic equation. Again for every point (x,y) in the xy-plane we obtain three intersections of l, the line through (x,y) parallel to the z-axis, whose z-coordinates are precisely the three roots of the cubic equation (5). Interpreted as a surface in a Cartesian space (5) represents a ruled cubic surface with the line at infinity of the xy-plane as the nodal or double line. The discriminant

$$4x^3/27+y=0 \tag{6}$$

of (5) is a cubical cylinder with a plane cuspidal cubic in the xy-plane as a base. The lines l parallel to the z-axis on the convex side of the discriminant cylinder cut the cubic surface (5) in only one real and two imaginary points. The generatrices of (6) are tangent to (5) and consequently cut (5) in two coincident and one other distinct point.

The cuspidal generatrix of the cylinder has three coincident points in common with the cubic surface.

Thus these models illustrate in a beautiful and extremely simple manner the various cases for the roots of quadratic and cubic equations of one unknown z. These models may be seen in the extensive collection of mathematical models in the University of Illinois.

THE STRAIGHT LINE INDISPENSABLE

Straight lines do not exist in reality. How rough are the edges of the straightest rulers, and how rugged are the straightest lines drawn with instruments of precision, if measured by the standard of mathematical straightness! And if we consider the paths of motion, be they of chemical atoms or terrestrial or celestial bodies, we shall always find them to be curves of high complexity. Nevertheless the idea of the straight line is justified by experience in so far as it helps us to analyze the complex curves into their elementary factors, no one of which is truly straight; but each one of which, when we go to the end of our analysis, can be represented as a straight line. Judging from the experience we have of moving bodies, we cannot doubt that if the sun's attraction of the earth (as well as that of all other celestial bodies) could be annihilated, the earth would fly off into space in a straight line. Thus the mud on carriage wheels, when spurting off, and the pebbles that are thrown with a sling, are flying in a tangential direction which would be absolutely straight were it not for the interference of the gravity of the earth, which is constantly asserting itself and modifies the straightest line into a curve.—From Paul Carus' "The Foundations of Mathematics."

The Two Wentworths

By Harry Gwinner
University of Maryland

The popularity of the Wentworth Mathematical Series and of the Wentworth-Smith Mathematical Series leads the writer to believe that some information relative to the Wentworths will be of deep interest to readers of the Magazine.

I made an attempt to ascertain the number of copies sold of the texts published by them. Though my effort was unsuccessful, I am firmly convinced that their sales have exceeded those of other similar texts.

George A. Wentworth

George A. Wentworth was born at Wakefield, New Hampshire, in July, 1835. The district school and the Wakefield Academy were his first schools. He then went to Phillips Academy, at Exeter, and finally entered the Sophomore class at Harvard College, from which he was graduated in 1858.

In April before graduation he was called to Phillips Academy, where he was professor of ancient languages for one year. He then became professor of mathematics in the same institution. In 1891 Professor Wentworth resigned his chair at Phillips Exeter, and spent a year in foreign travel. He died in 1906.

George Wentworth

George Wentworth was born in Exeter, New Hampshire, on January 8th, 1868, the son of George A. Wentworth, author of many well-known mathematics textbooks. He studied at Phillips Exeter Academy and at Harvard University. He then engaged in the railroad business in Minnesota; and from this he was soon promoted to the headship of one of the most important accounting departments in a great railroad system. Here he made a name for himself as an unusually expert and rapid accountant. He resigned this position to become associated with his father in the Wentworth Mathematical Series.

He was a careful student of educational affairs in schools of all grades and gave most serious attention to the individual features of many schoolrooms in order that he might adapt the Wentworth texts to the varying conditions of modern schools. His intimate knowledge of school work was derived in part from many years of service on the School Committee at Exeter, in which capacity he was called upon to perform many of the duties usually assigned to a superintendent of schools.

A few years after his father's death, he collaborated with David Eugene Smith in the Wentworth-Smith Mathematical Series. He died suddenly on August 26th, 1921.

The Teacher's Department

Edited by

JOSEPH SEIDLIN AND W. PAUL WEBBER

MATHEMATICIANS, RIGHT OR LEFT?

Before an audience of mathematicians one can scarcely have the temerity to propound the query "Mathematicians, right or left?" A unanimous reply, predetermined by years of association with the Queen of the Sciences is sure to follow: "Mathematicians are always right." Little does it matter what the alternative may be. Then, too, in another connotation of the terms, the mathematician is likely to be found on the Right or Conservative side due to his training to question every statement or claim until it has been proved. I believe it was a mathematician who, upon glancing at a flock of sheep, remarked that they had been sheared "on this side."

But to come to the subject that I wish to discuss! What attitude shall we teachers of mathematics take toward the question of alteration of College Entrance Requirements? Sometimes in the dim past, the present form of requirement for College Entrance was formulated and has stood for many years practically unchanged. Whenever any change has been made, whether in the certificate plan or the examination plan, it has been in the direction of more specification rather than less. Just how the "ideal" combination of required units was arrived at I cannot say but it would appear that in the determining group there must have been a liberal sprinkling of representatives of the "exact" sciences.

Even with the development of the Junior High School plan of organization there has been practically no trend toward changing requirements. I presume that if the early division of the public school curriculum had been six years in the elementary school and six years in high school we would have in our college entrance requirements the specification of twenty-four units instead of the favored sixteen. If not, why do we still cling to the plan of requiring sixteen units (or fifteen) when in many schools twelve only of these are carried in the High School and the other four have to be lifted from the Junior High School curriculum? This anomaly is used only to illustrate the static condition of entrance requirements in Michigan colleges. Otherwise it is not germane to text of my argument.

For a few years the Michigan Education Association has had a committee working on a proposed revision of College Entrance Re-

quirements and a little while back a tentative new setup was proposed. The most revolutionary part of this proposal was that colleges be asked to accept "two units in mathematics or in a foreign language" instead of "two units in mathematics" for entrance of certain types of students. It was to be understood that this would be done only in cases where the student carried with him a most glowing recommendation from his principal and an affidavit that the substitution of foreign language for mathematics was the only academic crime he had committed. I cannot say definitely what has become of the recommendations of this committee but they must have met strong opposition somewhere and have been shelved for presentation at a more favorable time. And so we go on saying to every high school student, even though his aim in life may be to enter the diplomatic service or to become a great artist, a musician or just a cultured gentleman: "You must make room for a year of Algebra and a year of Geometry at the sacrifice of more History, Language, Economics, Sociology or other subjects that might really play a part in a unified program of studies."

This conservative attitude on the part of those responsible for our admission requirements seems to be nothing more than a hangover of the "urge of primitive man to require each member of the tribe to conform and to carry on as the experience of the fathers has dictated." I met this same tendency recently in dealing with our student interfraternity Council. The question of the abolition of "Hell Week" was up for discussion. What justification could the fraternity men give for keeping the neophytes on the go day and night for three days? Taking their cue, one can imagine, from committees on admissions, many of these young men ardently defended the thesis that until the pledges were completely worn down physically their character could not be determined nor could the fraternity decide whether they had the qualities that make up good fraternity men. Any young man who "could not take it" was asked to turn in his pledge pin. I rather suspect that my auditors would decide almost unanimously that a young man with the courage to stand out against such requirements would be worth more to a fraternity than all those who submit. And in this case I see considerable cause for hope. There are *students* on both sides of the arguments and I feel sure they are going to come together with the result that the barbaric initiation ceremony will be modernized. Is it too much to hope with college authorities on one side and potential college students on the other that outworn "initiation ceremonies" incident to College Admission may be modernized? Here, too, there is reason to hope that we may soon emerge from the Stygian night. Progress is being made in many states but so far as I can observe

Michigan colleges are playing the role of the tortoise which moves along in its hard shell, closes up safely inside when something new and strange appears and then thrusts out a long neck and four short legs and waddles on after all danger is past. Incidentally, this creature has been much glorified for winning a certain race ages ago but I don't recall that he has ever done anything since.

What progress is being made away from present specified entrance requirements? For those who are not familiar with it let me sketch the broad provision of the Cooperative Experiment of the Progressive Education Association. First "there shall be selected a number of schools which shall be free for a period of five to eight years to experiment with the curriculum of college candidates making these curricula sufficiently flexible to allow of adaptation to the individual student's needs and interests."

Second "the school and the college will cooperate in guiding the student through his secondary school so that he himself may get a view of his problem."

Third "the college shall receive, first of all, a recommendation from the principal of the school to the effect that the student has the capacity to do college work; that he has well-defined, intellectual interests and purposes; and, finally, that he is prepared to work in one or more fields in which the college is giving instruction. Then, in addition to that, the college will receive a cumulative record, detailed and comprehensive, following the student through his four years' career, giving his intellectual developments and behavior in general. This record will include the results of such tests and examinations as the directors feel will be illuminating."

Fourth: The colleges participating in the experiment shall admit or reject students on the basis of the records furnished rather than on the basis of specific entrance requirements.

Already about one-third of the colleges of the country have officially agreed to participate in this experiment.

The results of this experiment will be watched carefully by all participating schools. The Progressive Education Association will, of course, study the results and make them available to others. It will take the results of a study such as this conducted on a national basis to convince many of the educators of the country that admission requirements may safely be modified.

But in the meantime progressive colleges are making far-reaching changes in their requirements which may effect a complete revolution in relations between secondary schools and colleges even before the results of the Progressive Educational Association experiment become

available. Perhaps the most outstanding of these centers around the work of the Cooperative Test Service of the General Education Board. By use of carefully graded tests it is possible to build a cumulative record of the student's growth throughout his secondary school experience and have this record comparable with similar records from other schools. The difficulty with present certificates of record from secondary schools and colleges is pointed out by W. G. Learned of the Carnegie Foundation when he says, "Hence, when we are offered a pupil's record, even of A's and B's, in a series of scattered and hermetically sealed courses that can have presented but a minimum of inducement to genuine thinking, apart from teacher, text, and credit, we are almost as much in the dark as to the quality of that pupil's present working outfit as we were before. We know too well that, up to the point to which education has been carried under such circumstances, A's and B's can be acquired by minds with no lasting affinity whatever for the real values of the knowledge that has been surveyed. We may be dealing merely with good, willing conformists, working vigorously for the prestige with school and home which is easy for them to win and which they have been permitted to substitute for an intellectual purpose. The school certainly makes no demands that such motives cannot amply satisfy.

"The second point of weakness is one that still further vitiates the confidence already undermined by a dispersed and static curriculum. It is the purely subjective character of all the judgments expressed on the record. Not only do we have the pupil measured solely on his reactions between the beginning and end of short, disconnected courses in new stuff, but the measure is taken privately and arbitrarily by the teacher of a relatively small group who rates his pupils on the way he thinks they have responded to tasks that he has himself chosen and administered. Knowledge of the subject, experience with pupils, opportunity to judge, health, personal likes and dislikes, school promotion policies—all count as concealed variables in his verdict. Local standards of school work, levels of pupil mentality or background, and relations with higher institutions likewise sharply affect the gradings. Our first experience in testing high school seniors in Pennsylvania brought out the fact that in some large city schools operating under apparently normal conditions groups of pupils individually rated D in English by their teachers would secure higher average scores in a standard English test than groups individually rated A in other schools. We found one case where the average of all the C pupils was above that of all the A pupils in the same school, due to the simple reason that, after sectioning the class on the basis

of tested ability, the principal had entrusted the clever sections to a young and discriminating teacher who rarely gave A's and the other group to a sympathetic veteran who always "marked on effort" and therefore rarely gave any other grade."

A cumulative record of the pupil's secondary school work and attitudes would reveal very definitely his fitness for further training and would point the way toward vocational guidance. At the present time pupils have to concentrate upon "getting into college" rather than upon working out a course of study that will contribute most to a studied plan of education.

Let me quote from an address of Dean J. B. Johnston of the University of Minnesota: "Instead of the intellectually and morally degrading regulations which have been maintained by the faculties in most colleges, I would have the college:

(a) Admit any person who gives evidence of mental ability, interests, and energy, of habits of application and concentration, and of the possession of sufficient tools such as vocabulary and speed and intelligence in reading to handle studies of college grade;

(b) Give him an opportunity to get up studies of secondary grade which he may need but give him no credit for such studies;

(c) Aid the student to carry out his plans and give him as long or short a time as may be needed to educate himself, provided that he is doing work of college grade satisfactorily;

(d) Pay no attention to high school units for admission or to courses and credits in college or to time of residence as claims for degree;

(e) Set forth as the requirements for a degree

1. An acceptable objective and an intelligent plan for its achievement submitted by the student at an appropriate stage and approved by a duly constituted faculty adviser;
2. Interest and energy in studies and other educational activities called for in the plan;
3. Evidences of the development of intellectual power, of the plan being successfully carried out and the objective being attained; such evidences to be given through comprehensive examinations adapted on the one hand to the nature of the objective and on the other hand to the characteristics of the students.

Immediately it will be evident that if colleges generally operated along these lines, there could be no objection to accepting graduation from a secondary school as part of the evidence that the student possesses the minimum prerequisites for college work. Personally, I should

be much more interested in the evidence furnished by the experimental high school curriculum which has grown out of the Pennsylvania Study or that which is now being proposed by the Progressive Education Association."

The plan of the Progressive Education Association referred to previously in this paper is contained in recommendation A, the specific provisions of which are that to be admitted to college a student should (1) be possessed of the requisite general intelligence to carry on college work creditably; (2) have well-defined serious interests and purposes; and (3) have demonstrated ability to work successfully in one or more fields of study in which the college offers instruction.

If such a plan as this were carried out, our traditional two units of mathematics, two units of a foreign language, etc. would have to go. When this occurs in Michigan I shall not be among the mourners. Already Iowa, Ohio, Minnesota, Wisconsin and Kentucky have dropped the plan of specific entrance requirements and are admitting students on the basis of their school history and cumulative record. The University of Buffalo has dropped specific requirements in language and mathematics. Stanford and Chicago have specified requirements in English only. We in Michigan are now a sort of a backward island separated from the progressive mainland by a sea of conservatism that these progressive ideas seem not to have been able to cross.

Recently, however, I have been much impressed with the fact that the young folk in the schools are rebelling against the situation and are calling for modifications. If they join with the secondary school authorities in the effort to effect a more reasonable plan of college entrance the colleges might as well surrender. No turbulent sea will stave off the attack.

In the joint meeting recently closed in Cleveland of the Progressive Education Association and the Department of Superintendence of the National Educational Association, Professor Briggs of Columbia University, remarked, as reported in *Time*: "No credits are so frozen as many that are given in High Schools and Colleges. The facts are a professional scandal" and Professor John Dewey added, "Now, if ever, is the time for educational change. Today things could be proposed that could not have been a few years ago, and perhaps could not be a few years hence when general conditions may be more static."

In better times we are engaged in pushing out the boundaries of our institutions. Economic conditions resulting in curtailed budgets make extensions of this kind impossible. Our field for study, then.

has a definite horizon. A carefully planned introspective survey will surely lead to a more definitely integrated program of secondary school and college work, resulting in elision of the end of the secondary school with the beginning of college work. The only way to accomplish this is by material modification of present College Entrance Requirements. —DEAN C. H. EMMONS, *Michigan State College.*

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*HOW TO LOOK FOR THE SOLUTION

1. Understand the question.
2. Find a path leading from the unknown to the data, passing if necessary through several intermediate problems. (Analysis)
3. "Mettre en oeuvre." Execute your plan. (Synthesis)
4. Check and criticize.

I

What is it about? What is given? What is sought?

Do the data determine the unknown? Are they sufficient, insufficient, more than sufficient?

Can I put the question in any other way?

Can I relate the problem to another which I know already?

Can I relate the problem to another whose solution is simpler?

Or even to some other whose solution is immediate?

Have I already taken account of all the data?

These questions are to be repeated each time one is brought to a halt, and for each intermediate problem. One more important question: Have I already taken account of all the data?

II

Formulate the relation or relations between the unknown and the data.

Transform the unknown elements trying to introduce new unknowns nearer to the data.

Transform the given elements trying to deduce from them new data nearer to that which is sought.

Solve only a part of the problem.

Fulfill only a part of the conditions: what room for variation have you introduced by abandoning the other part? (Geometric loci)

Generalize—Specialize—Use Analogies.

III

Watch your steps and admit "only what one can see with evidence or deduce with certainty." (Descartes)

Substitute the defining facts in the place of the words defined. (Pascal)

IV

Is the result plausible? Why so?
In what way can I verify it?
Is there some other path leading to the result?
Is there some more direct path?
What other results would one be able to obtain in the same way?
Remember the four steps:

UNDERSTAND, TAKE APART, PUT TOGETHER, CHECK.

The mathematics teacher's most precious contribution to the general culture of his pupil is the formation within him of a facility and independence of reasoning in the solution of problems.

Three things the teacher must know: schoolboy psychology, the foundation and background of *method*, and how much help is enough. Three things he should do: give the pupil models to follow, emphasize those things which are worthy of being imitated, confirm useful habits of mind by designing suitable exercises. —G. POLYA.

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*Dear Professor Sanders:

Your excellent editorial in the January NEWS LETTER makes me believe that you would be interested in the enclosed work of Polya's.

These notes of a conference held at Berne, Switzerland, in October, 1931, by Professor G. Polya of the Federal Institute of Technology, Zurich, have been translated from Vol. XXX of *L'enseignement Mathematique*. This is the Polya who, along with Szego, is author of two collections of problems in higher analysis which have been published in the Courant series.

Sincerely yours,
NORMAN ANNING
University of Michigan.

February 16, 1934

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*Dear Professor Sanders:

I will be glad if you print in the NEWS LETTER the resume of my Berne lecture, translated by Professor Norman Anning.

There are a few expressions in Professor Anning's translation which, in my opinion, don't quite well fit the intended meaning. I enclose a sheet with proposals how to change them.

A more complete account of my lecture than that in the *l'Enseignement Mathematique*, used by Professor Anning, appeared in German, in the *Zeitschrift fur mathematischen und naturwissenschaftlichen Unterrischt* vol. 63 (1932) p. 159-169. I send you a copy.

Yours very truly,
G. POLYA

Dunantstrassea 4
Zurich 7
June 13, 1934.

Notes and News

Edited by

I. MAIZLISH

At the Rice Institute, Dr. C. H. Dix, Instructor in Mathematics, resigned during the autumn to accept a position with the Humble Oil and Refining Company. Mr. Floyd E. Ulrich of Schenectady, N. Y., formerly on the teaching staff of Union College, was appointed Instructor in Mathematics.

The majority of the programs of the semi-monthly meetings of the Alpha of Oklahoma chapter of Pi Mu Epsilon, at the University of Oklahoma, for last year dealt with the study of the development of the calculus and its inventors. The general theme of this year's study is an elementary theory of invariants and the relation of invariants to the solution of cubic and quartic equations. The program is in the form of papers presented by students at the regular club meetings, these papers being the result of investigation on the part of the student under the direction of the director of the organization or some other member of the faculty. The club has, for a number of years, sponsored various projects for the benefit of both organization and department. Last year, a French and a German dictionary were purchased and presented to the departmental library. The club also bought pictures of six outstanding pioneers in the field of mathematics, had them framed and hung on the walls of the mathematics classrooms. This report was sent in by Mary Oilla Tappan, secretary of the organization.

THE EDITOR INVITES ALL OTHER CHAPTERS OF PI MU EPSILON TO SEND HIM ANY ITEMS OF INTEREST.

Annually since 1911 (with the exception of the session 1925-26) the John H. French medal has been awarded to the cadet who obtained first standing in pure and applied mathematics at the Virginia Military Institute. The competition for the medal has just been widened to include the liberal arts and chemistry students, all of whom take analytic geometry and calculus in common with the engineering students. At the same time extra weight is given to the grades obtained in the more advanced courses to encourage study beyond the first course in calculus.

The scientists of Louisiana and Mississippi will gather at Louisiana College, Pineville, on the last week-end in March. The occasion is the

annual meeting of three organizations which will hold joint conventions; the associations are the Louisiana-Mississippi section of the Mathematical Association of America, the Council of Mathematics Teachers, and the Louisiana Academy of Sciences which is affiliated with the American Association for the Advancement of Science. The convention will open with a public lecture on Thursday night, March 28, and will close Saturday noon. More than fifty papers will be presented before the various sections of the organizations during Friday and Saturday. One high point of the convention will be the public lecture by Dr. H. J. Ettlinger of the University of Texas on Friday night. Arrangements are being made by the hosts at Pineville for a special dinner for the guests. Plans are also being made for those who so desire to visit local places of interest. Anyone desiring to present a paper at the convention, or desiring to get information may write to Prof. T. A. Bickerstaff, University, Miss., or to Dr. John B. Entrikin, Centenary College, Shreveport, La.

Chairman T. A. Bickerstaff announces the following almost complete program for the meeting of the Louisiana-Mississippi section of the M. A. of A. to be held at Louisiana College, Pineville, on March 29th and 30th:

"The Geometry of the Complex Triangle" by Professor B. E. Mitchell, Millsaps College.

"Some Problems Solved by Heaviside's Directional Calculus" by Professor J. F. Thomson, Tulane University.

"A Construction for the Tangents at the Nodes of the Rational Plane Quintic" by Professor Elsie J. McFarland, Jones County (Miss.) Junior College.

"A Foundation for Riemannian Geometry" by Professor H. L. Smith, Louisiana State University.

"Non-Unique Solutions of Differential Equations" by Professor H. J. Ettlinger, University of Texas.

Meetings in conjunction with this group will be the division of the National Council for Teachers of Mathematics for the same territory and also the Louisiana Academy of Sciences.

Professor H. J. Ettlinger serving as guest speaker will also deliver a public address.

Everyone interested in science or mathematics is invited and urged to attend these meetings.

The contests of the Southern Intercollegiate Mathematics Association have been in progress for the last few weeks. Examinations are yet to be given in Analytics, Calculus and one covering the entire field covered by the separate examinations. This year, the Southern Intercollegiate Mathematics Association will hold its annual meeting in the early part of May at Centenary College, Shreveport. Colleges desiring to join the Association, which has been endorsed by some of the country's leading mathematicians, are urged to write to the Editor for further information.

THE DISCIPLINARY VALUE OF MATHEMATICS

The principal value of mathematical study arises from the fact that it exercises the *reasoning power more*, and claims *from the memory less*, than any other secondary school subject. The study of mathematics should result in the development of *power*, rather than in the acquisition of facts. Not he who knows a great many mathematical facts is a good mathematician, but he who can apply these facts intelligently, who can discover facts that are new to him, and who can reconstruct those which he has forgotten.

It is power and not knowledge that furnishes the true test of mathematical ability, and if the power is acquired, then—and only then—will the knowledge follow as a natural consequence. Mathematical instruction in a secondary school is—or rather should be—principally a systematic training in reasoning, and not an imparting of information.

Of course similar claims are made for nearly all other subjects, but a closer inquiry will show that for mathematics they are really justified. The reasoning in mathematical work is of a peculiar kind, possessing characteristics that make it especially fitted for training the minds of the students. Some of these characteristics are the following:

1. *Simplicity.*
2. *Accuracy.*
3. *Certainty of results.*
4. *Originality.*
5. *Similarity to the reasoning of life.*
6. *Amount of reasoning.*

—From Arthur Schultze's "The Teaching of Mathematics in Secondary Schools."

Problem Department

Edited by

T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 73. Proposed by Alexander Y. Boldyreff, University of Arizona.

Prove that

$$\frac{\sqrt{20+\sqrt{20+\sqrt{20+\ldots \text{ to } \infty}}}+\sqrt{12+\sqrt{12+\sqrt{12+\ldots \text{ to } \infty}}}}{\sqrt{6+\sqrt{6+\sqrt{6+\ldots \text{ to } \infty}}}}=3.$$

Solved by Dewey C. Duncan, University of California.

Let $\sqrt{n(n+1)+\sqrt{n(n+1)+\sqrt{n(n+1)+\ldots \text{ to } \infty}}}=x.$

Squaring, one obtains

$$n(n+1)+\sqrt{n(n+1)+\sqrt{n(n+1)+\ldots \text{ to } \infty}}=x^2.$$

i. e., $n(n+1)+x=x^2$; whence $x=(n+1)$. [$x=-n$, extraneous].

Therefore

$$\sqrt{n(n+1)+\sqrt{n(n+1)+\sqrt{n(n+1)+\ldots \text{ to } \infty}}}=(n+1).$$

Accordingly

$$\sqrt{20+\sqrt{20+\sqrt{20+\ldots \text{ to } \infty}}}\;=5$$

$$\sqrt{12+\sqrt{12+\sqrt{12+\ldots \text{ to } \infty}}}\;=4$$

$$\sqrt{6+\sqrt{6+\sqrt{6+\ldots \text{ to } \infty}}}\;=3;$$

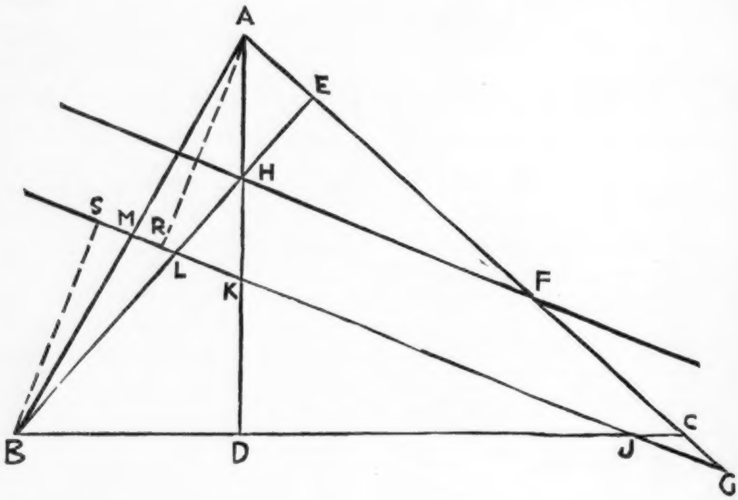
whence the theorem is seen to follow.

Also solved by David F. Barrow, Athens, Ga.; R. A. Miller, University of Mississippi; J. R. K. Stauffer, Laurel, Delaware; V. C. Harris, Northwestern University, and the proposer.

No. 76. Proposed by Walter B. Clarke.

Let A, B, and H be any two vertices and the orthocenter of a triangle. From midpoint of AB draw a line parallel to the bisector of angle AHB. Show that this line cuts the perimeter of triangle into two equal parts.

Solved by Henry Schroeder, L. P. I., Ruston, La.



$\angle JKD = \angle FHK = \angle FHE$

$\angle DJK = \angle HFE = \angle JGG = \angle CJG$

Triangle JGC is isosceles.

$JC = GC$

$AC + JC = AG$

AR and BS are drawn perpendicular to GM.

$AR = BS$

$\angle BLS = \angle LKH$

Triangles BLS and RKA are congruent.

$BL = AK$

Triangles BJL and AKG are congruent.

$BJ = AG = AC + JC$

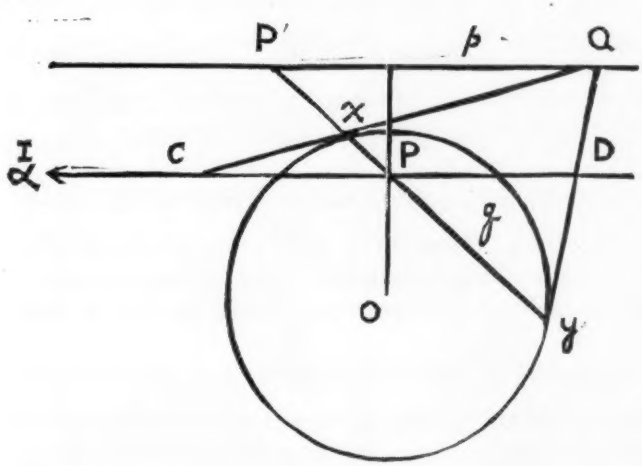
Q. E. D.

While solutions have been published for No. 52, (April, 1934 and December, 1934) the following different approach is offered by T. E. Raiford, University of Michigan.

Proposed by H. T. R. Aude, Colgate University.

Given a circle with center at O and a point P, not on the circle. A line is drawn through P cutting the circle at two points at which the tangents are drawn. These meet the line through P which is perpendicular to PO in the points C and D. Are PC and PD equal in length?

Solution:



From any point P not on the circle O draw a secant line q cutting the circle in the points x and y. Tangents at x and y meet in some point Q the polar of q. The polar p of P passes through Q and is

perpendicular to the line PO. The intersection P′ of p and q harmonically separates x and y from P. Projecting the harmonic set PxP′y from center Q upon a line u perpendicular to PO at P gives the harmonic set $PCI_\alpha D$ since u and p are parallel. P is therefore the midpoint of CD.

LATE SOLUTIONS

Nos. 71, 72 and 74 by J. R. K. Stauffer, Laurel, Del.

By Alexander W. Boldyreff, No. 72, using vector notation, and No. 74.

No. 74 by Corwin D. Moore, student, St. Joseph Junior College, Oregon, Missouri.

No. 65 by Rev. F. M. Kenny, Malone, N. Y.

PROBLEMS FOR SOLUTION

No. 78. Proposed by Vincent C. Harris, Northwestern University.

Find the value of

$$\sqrt{12+\sqrt{48+\sqrt{768+\sqrt{196608+\cdots\cdots}}}}$$

No. 79. Proposed by Norman Anning, University of Michigan.

Two parabolas are in the same plane and their axes are at right angles. Prove that their intersections are concyclic.

No. 80. Proposed by Donat Kazarinoff, University of Michigan.

Consider the sections of a paraboloid of revolution by planes through its focus. Show that the focus of the surface is a focus of every section and the corresponding directrices lie in a plane.

No. 81. Proposed by Walter B. Clarke, San Jose, California.

Through point P inside or outside the triangle ABC, draw a set of cevians cutting the sides (prolonged if necessary) at D, E, and F respectively. Transpose the segments of each side obtaining points D′ E′ F′ respectively. AD′, BE′, and CF′ will be concurrent at P′. What is the locus of P so that PP′ will be parallel to one of the sides of the triangle?

No. 82. Proposed by Dewey C. Duncan, University of California.

A spider is resting at the top of a pyramid whose base is a rectangle $7\frac{1}{2}$ by 6 feet. The top of the pyramid is four feet above the center of the base. The spider wishes to make a complete journey in the shortest possible way, traversing each of the eight edges of the pyramid at least once and ending his course at the top. Calculate the length of his journey. In how many different ways can he traverse this shortest course?

No 83. Proposed by William E. Byrne, Virginia Military Institute.

Find the limit of θ as $h \to 0$ in the theorem of the mean

$$f(a+h)=f(a)+h\,f'(a+\theta h)$$

when $f''(x)$ is continuous. What happens when $f''(x)=0$ identically? Is the limit the same when $f''(a)=0$, with $f''(x)$ not identically zero? Calculate the limit of θ under the last named conditions when $f'''(a)\neq 0$. A check is afforded by $f(x)=A+Bx+Cx^3$, $c\neq 0$; $a=0$.

No. 84 Proposed by Walter B. Clarke, San Jose, Calif.

Given two concentric circles O_1 and O_2, the outer one being O_1. Required to find circle O_3 with center on O_2 and such that if from any point P on O_1 tangents are drawn to O_3 and cutting O_1 at Q and S, the line QS will also be tangent to O_3.

No. 85 Proposed by Norman Anning, University of Michigan.

Simplify $1+2\cos 2A+2\cos 4A+\;.\;.\;.\;.\;+2\cos 2nA$; n, any positive integer.

No. 86 Proposed by Henry Schroeder, Louisiana Polytechnic Institute.

Using compasses only, find the center of a circle having given the circumference.

No. 87 Proposed by F. A. Rickey, L. S. U.

A diameter AB of a certain circle is extended through A to C so that the length of AC is 3 inches. At B a line segment, BD, is drawn perpendicular to AB. The length of BD is 9 inches. Determine the length of AB so that DC shall be tangent to the circle.

Can this problem be solved without the use of algebraic equations of degree higher than second?

Book Reviews

Edited by
P. K. SMITH

Higher Mathematics for Engineers and Physicists. By Ivan S. Sokolnikoff and Elizabeth S. Sokolnikoff. New York, McGraw-Hill Book Company, 1934. 482 xiii pages. \$4.00.

The subject matter covered in this book is indicated sufficiently by the fifteen chapter headings: Elliptic integrals, solution of equations determinants and matrices, infinite series, multiple integrals, line integrals, improper integrals, ordinary integrals, ordinary differential equations, partial differential equations, vector analysis, probability, empirical formulas and curve fitting, conformal representation.

According to the statement of the authors in the preface: "too strenuous an insistance upon rigorous and formal presentation of mathematics to junior students in applied sciences is likely to cause more harm than good. The keynote of this book is practical utility, and the authors have given considerable thought to the selection of those topics which are of most frequent and immediate use in applied sciences . . ." How well is this aim attained? The writer of this review feels that the authors have succeeded in giving many interesting physical applications, which might well serve as a stimulus to further reading in mathematics, but that a student upon completion of the text would not have sufficient technique at this disposal to handle such applications independently and surely. After all the limitations of a theorem are indicated in the proof; even a brief outline of the proof would help in case the discussion itself is too complicated for an elementary course.

The presentation is as a whole very careful. In chapter I we have $(dx/dt)^2 = f(x)$. In solving for dx/dt the proper sign of the radical should be specified according to the initial conditions: why should the discussion be limited to motion to the right, i. e. $d\theta/dt > 0$ in the pendulum problem? If Chapter XII (Vector Analysis) had been placed earlier in the book several discussions (for instance, that of directional derivatives) could have been simplified. The discussion of the theory of small errors could have been amplified with profit to include the formula

$$\text{maximum}\left|\Delta f\right| = \left|\frac{\partial f}{dx}\right| \left|\Delta x\right| + \left|\frac{\partial f}{dy}\right| \cdot \left|\Delta y\right| \quad \text{approximately}$$

where f is a function of x and y. On page 183 (Chapter VIII) the *exterior normal* of an element of surface dS is not defined. The rule given on page 233 for exact differential equations may fail (see Cohen, *Differential Equations*, first edition, page 11). On page 330 it is stated that "in order to completely specify a vector, it is necessary to specify its magnitude and any two of its direction angles." The method indicated in the text determines only the *square* of the cosine of the third direction angle.

There is undoubtedly a demand for courses and texts between the first course of calculus and the advanced calculus as typified by Wilson's *Advanced Calculus*, and this book meets the demand for a survey course in this field quite well. It is hoped it will have the success it merits. There are very few misprints and not much duplication of previous courses, save perhaps in Chapter II, which is covered generally in algebra and elementary calculus.—WM. E. BYRNE.

Intermediate Algebra. By Aaron Freilich, Henry H. Shanholt, Joseph P. McCormack. Silver, Burdett and Company, New York, 1934.

This book is attractively bound. It contains 406 pages. The arrangement of the material is neat and orderly, and there is an unusual number of good figures and graphs.

Its contents are divided into thirteen chapters. The subject matter is sufficient for a satisfactory course in second year algebra. Preceding the index are a few supplementary pages devoted to helpful material for teachers and typical examination questions.

Included in each chapter are an appropriate historical sketch, exercises divided into three groups that make the course flexible and easily adapted to the individual differences of students, and a cumulative review.

An interesting feature of the text is the thought questions under the topic heading "Something to Think About." These questions are introduced after the development of a new idea, and the writer believes that such questions are splendid for stimulating original thinking.

Chapter VI on "Deriving and Using of Formulas" should serve to give the student a clear understanding of the formulas, and to develop his ability to summarize from his own observations.

The chapter on "Graphs and Functions" is one in which excellent application is made of the use of mathematics in science, business, industry and every day life.

True to the claims of the authors, major emphasis has been placed on those basic ideas which function most in life, such as the formula, graph, equation and problem. Sections of the book have been devoted to "A Study in Changes." These develop the function idea and show the relation of mathematics to life.

This text should be popular with teachers who wish to follow progressive tendencies in the teaching of high school mathematics. —HENRY F. SCHROEDER.

NOTE ON INTRODUCTION OF AREA CONCEPT

When we introduce areas to children, *do* they comprehend what we mean? I have an idea they will be much more likely to do so, if the teacher have them cut out of cardboard quite a number of squares, one inch on a side, and actually use these as measuring instruments, just as their foot rulers. Have them see how many of these squares it takes to *cover* a book, or a notebook, or rectangles and squares drawn on paper. Then do the same for square foot; probably have each child make and own only one such square, cut from cardboard, and with the supply of the entire class, take turns seeing how many of these squares are necessary to cover a desk, or table, or a large rectangle drawn on the floor.

Will these children have any trouble comprehending what we mean by a square inch and a square foot, or why the area of a rectangle is in square measure, the product of the base by the altitude? Will they not see how many squares they can place along the base, and how many rows there are, each containing the same number of squares?

In January, 1932, in an article in this journal a former city school Superintendent said one of his sixth grade teachers had tried for *three weeks* to teach her class to find the area of rectangles, and "that not one of them could do it." I believe that if those children had cut out squares (a square inch and square foot), and had actually *measured* with them, they would have comprehended the matter in three days, probably in less time.

It will be good motivation for discovering some other way of determining areas of parallellograms, triangles, trapezoids, and circles, for them to see by experiment that they can not tell how many of their "measuring squares" are necessary to cover these figures, since the squares do "not come out even" when placed on any other figure than square and rectangle. —LENA R. COLE, *Central Normal College, Danville, Indianna.*